

## Measurements of the $\pi^+p \rightarrow \pi^+p\pi^0$ Cross Section from 0.5 to 1.5 GeV/c

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We have measured with counters the cross section for  $\pi^+p \rightarrow \pi^+\pi^0p$  reaction from 0.5 to 1.5 GeV/c. This cross section increases rapidly between 600 and 900 MeV/c to reach 10 mb. Using our results we have also calculated the  $\pi^+p \rightarrow \pi^+\pi^+n$  cross section.

### I. INTRODUCTION

WHILE the mechanism of the  $\pi p$  scattering has been explained below 300 MeV by the static model of Chew and Low, at higher energies the situation is more complicated since, on one hand, a large number of partial waves may take part in the interaction and since, on the other hand, the role of inelastic channels becomes increasingly important.<sup>1</sup> Between 300 and 1500 MeV, the most important of these channels are the reactions of single-pion production. In view of the interest of these reactions and since only the reactions  $\pi^-p \rightarrow \pi^-\pi^+n$ <sup>2</sup> and  $\pi^-p \rightarrow \pi^0\pi^0n$ <sup>3</sup> have been studied systematically so far (others are known only for certain energy values from bubble chamber measurements), we undertook the study of single  $\pi^0$  production in the reactions  $\pi^\pm p \rightarrow \pi^\pm\pi^0p$ . Preliminary results were presented at the 1962 CERN Conference.<sup>4</sup> In the present paper, complete results are presented for the  $\pi^+p \rightarrow \pi^+p\pi^0$  reaction.<sup>5</sup>

We have measured the cross section for the  $\pi^+p \rightarrow \pi^+p\pi^0$  reaction from 0.5 to 1.5 GeV/c incident  $\pi^+$  momentum by counting the photons from the disintegration of the  $\pi^0$  mesons, using scintillation detectors with lead converters, in coincidence with the charged secondary particles. Special care was taken to make the detection efficiency independent of the spectrum and of the angular distribution of the  $\pi^0$ . The detection efficiency for a photon from a  $\pi^0$  disintegration

varied only from 0.67 to 0.78 for a variation of the  $\pi^0$  momentum between 0 and 1.5 GeV/c.<sup>6</sup>

The results of the measurement would be independent of the angular distribution of the photons if the photon detector fully surrounded the target. For the simplicity of construction, and since the angular distribution is independent of the azimuth angle  $\varphi$ , the counters did not cover the whole range of  $\varphi$ . But in order to make the results of the measurements independent of the angular distribution of the photons in the polar angle  $\theta$ , the photon detectors must cover a large fraction of the range of  $\theta$ .

Our setup was not capable of distinguishing between the different production schemes of the  $\pi^0$ , such as

$$\begin{aligned} \pi^+p &\rightarrow \pi^+p\pi^0 \\ &\pi^+p\pi^0\pi^0 \\ &\pi^+n\pi^+\pi^0. \end{aligned}$$

If  $m$   $\pi^0$  mesons are produced in a reaction, we shall count  $2m$  photons. We obtain thus a "weighted" cross section  $\sigma_{\gamma^0}$  of the photon production in which a channel with  $m\pi^0$  has a weight  $2m$ . Using the known results for the multiple production and for the other reactions producing photons, we corrected  $\sigma_{\gamma^0}$  to obtain the cross section for the reaction  $\pi^+p \rightarrow \pi^+p\pi^0$ .

### II. EXPERIMENTAL ARRANGEMENT

The experiment was carried out in the laboratory of the proton synchrotron Saturne at Saclay. A doubly analyzed secondary pion beam was used (Fig. 1). The particles emitted at an angle of  $18.5^\circ$  from the internal copper target were focused and momentum-analyzed by a quadrupole triplet and a bending magnet. In the plane of the intermediate focus, a collimator defined the momentum interval

$$\Delta P/P = \pm 2\%.$$

The second part of the beam transport system, a C-shaped magnet and another quadrupole triplet, had a dispersion which compensated that of the first part. The intensity of the  $\pi^+$  meson beam varied from 5000 to 1100  $\pi^+$ /cycle between 0.5 and 1.5 GeV/c. To reduce the multiple scattering of the particles the beam was transported in a vacuum pipe.

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<sup>1</sup> R. F. Peierls, Phys. Rev. **118**, 325 (1960). R. Ommes and G. Valladas, in *Proceedings of the Aix-en-Provence International Conference on Elementary Particles* (Centre d'Etudes Nucléaires de Saclay, Seine et Oise, 1961), Vol. I, p. 467.

<sup>2</sup> W. A. Perkins, J. C. Caris, R. W. Kenney, and V. Perez-Mendez, Phys. Rev. **118**, 1364 (1960); V. Alles-Borelli, S. Bergia, E. Perez Ferreira, and P. Waloschek, Nuovo Cimento **14**, 211 (1959); I. Derado and N. Schmitz, Phys. Rev. **118**, 309 (1960).

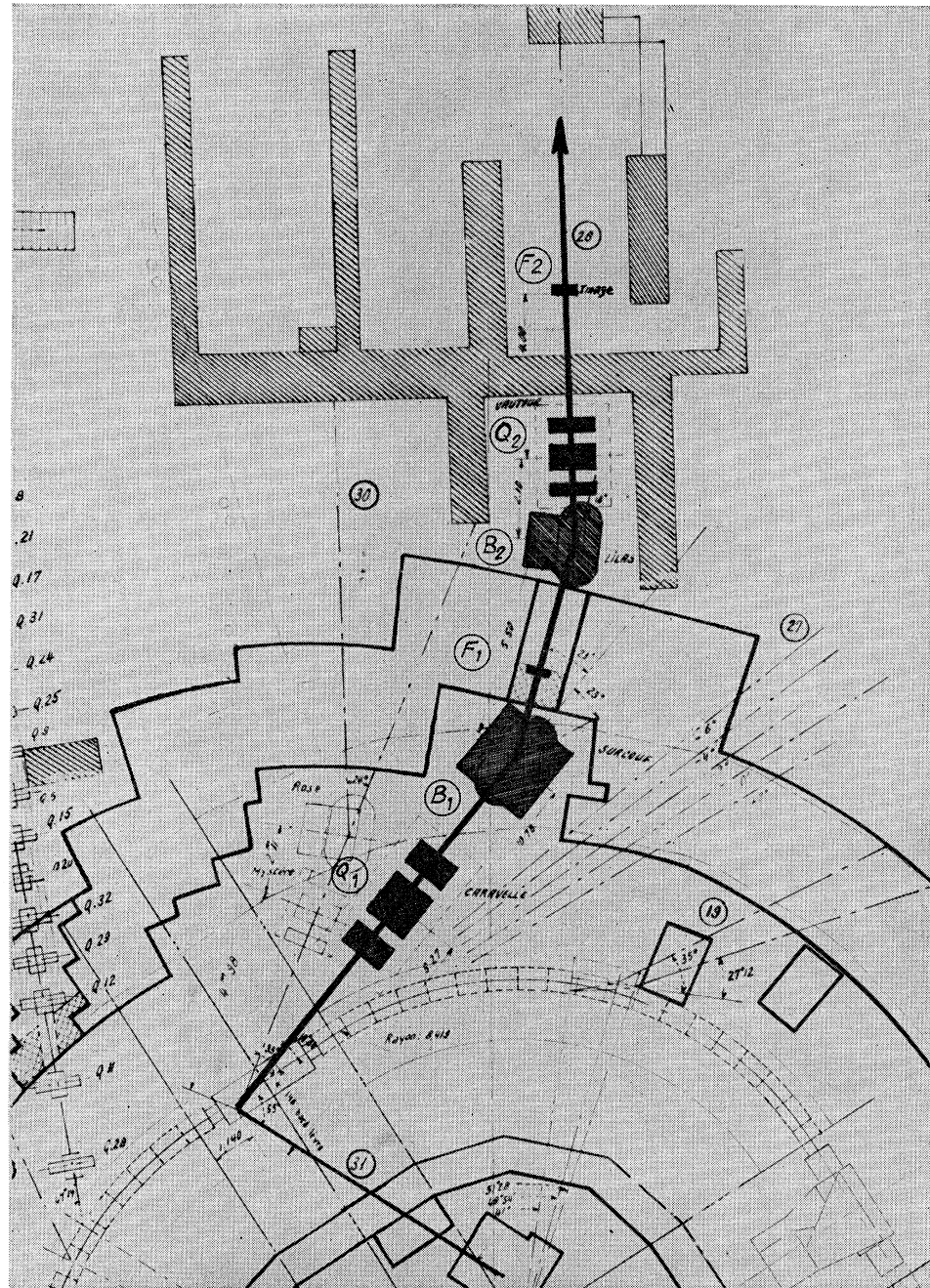
<sup>3</sup> J. C. Brisson, P. Falk-Vairant, J. P. Merlo, P. Sonderegger, R. Turlay, and G. Valladas, in *Proceedings of the Aix-en-Provence International Conference on Elementary Particles* (Centre d'Etudes Nucléaires de Saclay, Seine et Oise, 1961), Vol. I, p. 45; R. Turlay, thesis, Commissariat à l'Energie Atomique Report No. CEA-2136 (unpublished).

<sup>4</sup> J. F. Detoeuf, Y. Ducros, J. P. Merlo, A. Stirling, B. Thevenet, L. Van Rossum, and J. Zsembery, in *Proceedings of the International Conference on High-Energy Physics, CERN 1962* (CERN, Geneva, 1962), p. 7.

<sup>5</sup> The results concerning the reaction  $\pi^-p \rightarrow \pi^-p\pi^0$  have been published in a separate paper: Phys. Letters **8**, 74 (1964).

<sup>6</sup> B. Thevenet, thesis, Commissariat à l'Energie Atomique Report No. CEA-2306 (unpublished).

FIG. 1. Diagram of the beam support system.  $Q_{1,2}$ =quadrupole triplets,  $B_{1,2}$ =bending magnets,  $F_1$ =intermediate focus,  $F_2$ =final focus.



The protons present in the secondary beam were eliminated by the time-of-flight method, and the electrons were eliminated by using a Čerenkov gas counter in anticoincidence.<sup>7</sup> The contamination due to  $\mu$  mesons was measured using another gas Čerenkov counter<sup>8</sup> placed at the position of the hydrogen target.

<sup>7</sup> J. Sayag, J. Phys. Radium 23, 169A (1962).

<sup>8</sup> J. Duboc, J. Banaigs, and J. F. Detoeuf, J. Phys. Radium 22, 64A (1961).

In the range from 0.5 to 1.5 GeV/c this contamination varied between 12 and 3%.

The liquid-hydrogen target consisted of a Mylar bag 20  $\mu$  thick. The support and the thermal insulation of this bag were made of Styrofoam ( $\rho=5 \times 10^{-2}$  g/cm<sup>3</sup>). The effective length of the hydrogen was equal to 30 cm. When working with an empty target, a pool of liquid hydrogen maintained a constant temperature of the gaseous hydrogen.

The counter assembly consisted of three distinct

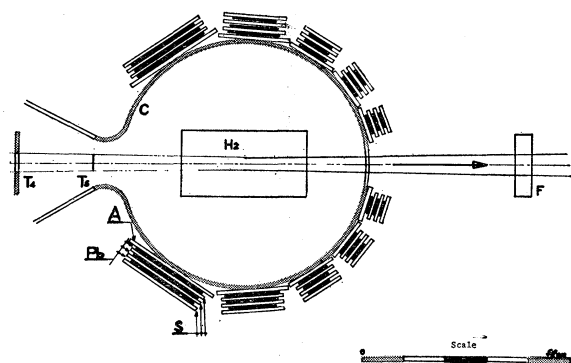


FIG. 2. Arrangement of the  $C$  counters and of the photon telescope.  $T_i$ =incident telescope,  $P$ =Čerenkov counter,  $C=4\pi$  scintillator,  $(AS)_i$ =photon telescope.

groups (Fig. 2): the incident telescope  $T$ , the counters surrounding the target  $C$ , and the photon telescopes  $\gamma$ .

The incident telescope, consisting of five scintillation counters and a Čerenkov counter in anticoincidence to eliminate electrons, defined the geometry of the incident beam and enabled us to separate the protons from the  $\pi^+$  by the time-of-flight method over a path of 8.80 m. A terminal anticoincidence counter  $F$  behind the target helped to reduce the counting rate. The target was surrounded by the counter group  $C$  consisting of six elements which formed a cylindrical box. The signals from the six scintillation counters were mixed in a mixer circuit MC, which gave an output signal whenever a charged secondary passed through the counter.

Two groups of five telescopes, for photon detection (Fig. 2), were placed symmetrically on either side of the target, in the horizontal plane. Each of these telescopes consisted of a scintillator  $A_i$  in anticoincidence, followed by a counter  $S_i$  composed of three

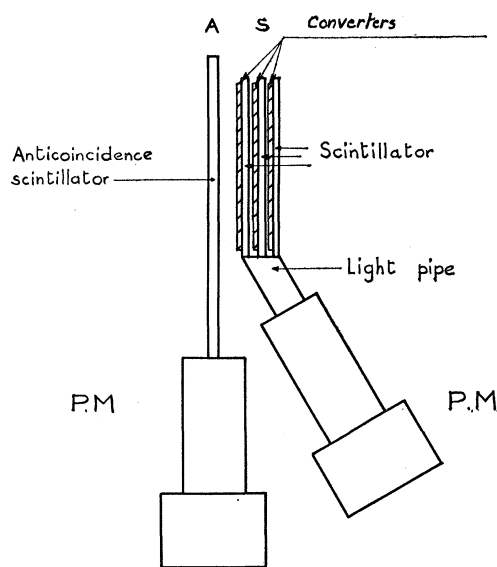


FIG. 3. Diagram of a photon detector.

scintillator sheets glued to the same light pipe (Fig. 3). It was possible to insert a lead or carbon converter in front of each sheet. The two groups of five telescopes, operating independently, were used to check the performance of the apparatus. Each group covered the angular interval from  $10$  to  $160^\circ$  in the laboratory system, the corresponding solid angle being equal to 1 sr. The dimensions of each telescope were such that it covered approximately the same solid angle in the center-of-mass system.

The measurements consisted in the identification and counting of the events in which an incident  $\pi$  meson interacted in the hydrogen, causing a simultaneous

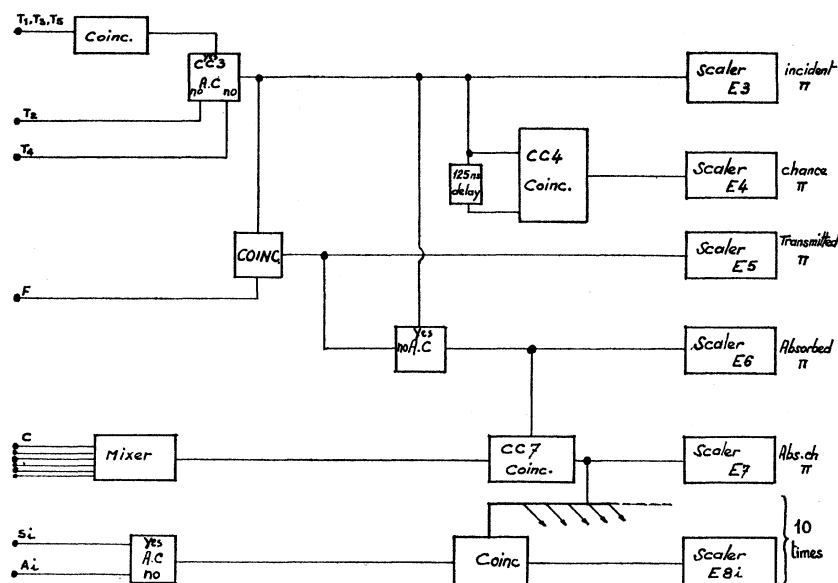
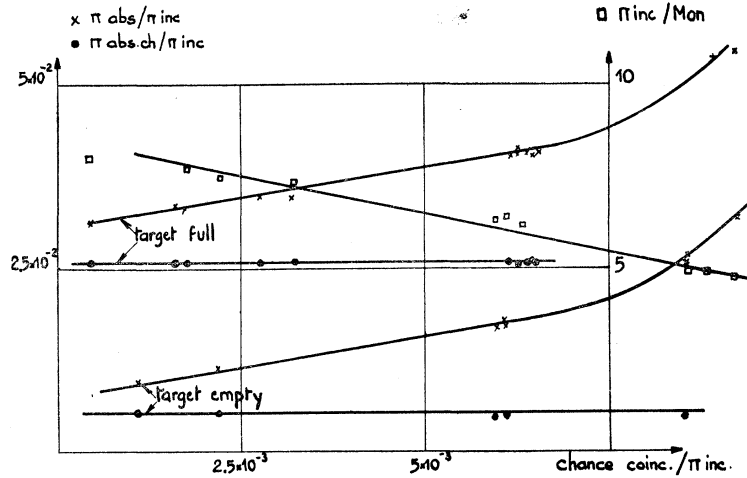


FIG. 4. Block diagram of the electronics.

FIG. 5. Counting-rate effects (measurement taken at 650 MeV/c).



emission of a charged secondary particle and of a photon. The different stages of the reaction identification are indicated in the block diagram in Fig. 4. The photomultipliers used were Radiotechnique Types 56 and 58 AVP, and the coincidence circuits were of the additive type with distributed parameters.<sup>9</sup>

To estimate the effects of chance coincidences and counting losses, we counted, using the circuit CC4, the coincidences between the fast output of CC3 and the same output delayed by 125 nsec. The delay was chosen according to the accelerating frequency of the protons in Saturne equal to 8 MHz. The counting rate  $E4$  of these "chance  $\pi$ " is an unknown function of the beam rate, tending to zero with decreasing beam rate. Measurements were taken at different rates, and the ratios  $R_i$  of the different scalers were plotted as a function of the counting rate of  $E4$ . It was then sufficient to extrapolate to zero the curves  $R_i = f(E4)$  to obtain their "true" value, at zero counting rate. By this method the chance coincidences, the counting losses, and any other rate effects are taken into account. At all beam energies we found the following typical phenomena (Fig. 5).

(1) The apparent ratio  $\pi$ -absorbed/ $\pi$ -incident increases with increasing beam rate. The curve is a straight line below a certain value of the counting rate, having the same slope for a full target and for an empty one. This enabled us to account for the effect simply by measuring both target full and target empty at the same rate.

(2) The circuits following CC7 do not exhibit any counting rate effect in the region under consideration. The above tests were carried out systematically, and in each case the highest beam intensity was used for which the above method did not yet reveal any counting rate effect.

Each measurement was carried out with and without hydrogen in the target. In either case the measurements were made with both lead and carbon absorbers. The lead absorbers were 4 mm thick; the efficiency of a telescope with three such sheets was of the order of 70% for photon energies between 100 and 1500 MeV. We took into account the fact that the lead could also cause the detection of neutrons produced, e.g., in the reaction  $\pi^+ p \rightarrow \pi^+ \pi^+ n$ . The absorbers of carbon with density 1.8, 6 mm thick, have the same efficiency for neutron detection as 4 mm sheets of lead (of the order of 3%) but only a 4 to 5% efficiency for the detection of photons. The calculations of the efficiency of lead and carbon for photons and neutrons are given in detail in Ref. 6.

For each value of the incident pion momentum we carried out four measurements: The measurements with the full target and lead absorbers (HPb), full target and carbon absorbers (HC), and the same measurements with the empty target (VPb and VC). The number of photons emitted from the hydrogen and

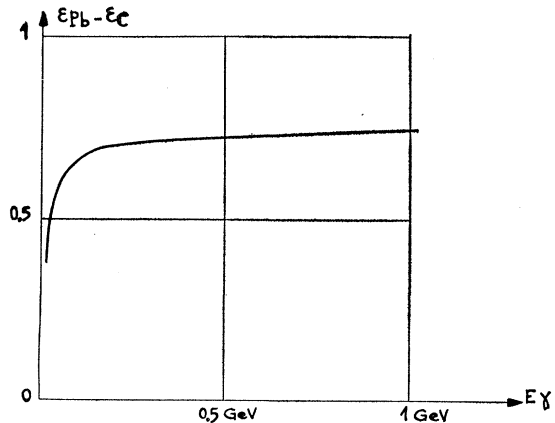


FIG. 6. Difference of the efficiencies of the lead and carbon converters for photon detection as a function of energy.

<sup>9</sup> J. C. Brisson, G. Valladas, and R. Van Zurk, *Proceedings International Symposium on Nuclear Electronics, Paris 1958*, (IAEA, Wien, 1959), p. 233.

detected with the efficiency Pb-C (Fig. 6) is then given by

$$N_\gamma = (HPb - VPb) - (HC - VC).$$

The counting rate  $N_\gamma^i$  of each telescope (i) was recorded separately on a scaler  $E8_i$  (see Fig. 4).

### III. EXPERIMENTAL RESULTS

#### A. Calculation of the Cross Section

The counting rates  $N_\gamma^i$  were first corrected for the  $\mu$  meson contamination  $C$ . If we denote by  $N_\gamma'^i$  the corrected counting rates, we have

$$N_\gamma'^i = N_\gamma^i / (1 - C).$$

The experimental data consisted of ten numbers  $N_\gamma'^i$ . A mean was formed of the counts of each pair of symmetrical telescopes (after their consistency had been checked):

$$G_\gamma^i = (N_\gamma'^i + N_\gamma'^{i+5}) / 2.$$

The relation between the five resulting values and the total cross section  $\sigma_\gamma$  for photon production was calculated, using an IBM-7090 computer, according to the following method: We assumed that the angular distribution of the  $\pi^0$  mesons in the center-of-mass system is given by the formula

$$\partial^2 \sigma_{\pi^0}^* / \partial \Omega^* \partial p^* = f(p^*) \sum_{l=0}^n a_l P_l(\cos \theta^*),$$

where  $p^*$  is the momentum, and  $\theta^*$  is the scattering angle of the  $\pi^0$  in the c.m. system. We assumed that the spectrum  $f(p^*)$  is the same for all angles in the c.m. system. Since the efficiency for detecting a photon is almost independent of the energy and the direction of the  $\pi^0$ , the use of an average spectrum is certainly a good approximation and by far sufficient in this calculation. We deduce the angular distribution of the photons detected in the laboratory<sup>10</sup> from the distribution

$$\left( \frac{d\sigma_\gamma}{d\Omega_\gamma} \right)_{\text{det}} = \frac{m_{\pi^0}^2}{2\pi\Gamma^2(1-B\cos\theta)^2} \sum_{l=0}^n a_l P_l(\cos\theta^*) \times \int_0^{p_{\text{max}}^*} f(p^*) dp^* \int_{-1}^{+1} \frac{\epsilon(E_\gamma) P_l(\cos\eta) d(\cos\eta)}{(E^* - p^* \cos\eta)^2},$$

where  $\theta$  is the angle of emission of the photon in the laboratory system,  $\eta$  is the angle between the direction of a decay photon and the  $\pi^0$  meson,  $Bc$  is the velocity of the c.m. system,  $m_{\pi^0}$  is the mass of the  $\pi^0$ ,

$$\begin{aligned} \cos\theta^* &= (B - \cos\theta) / (B \cos\theta - 1), \\ E^* &= (p^{*2} + m_{\pi^0}^2)^{1/2}, \\ \Gamma &= 1 / (1 - B^2)^{1/2}, \end{aligned}$$

and  $p_{\text{max}}^*$  is the maximum momentum of the  $\pi^0$  meson in the c.m. system.  $E_\gamma$  can be expressed as a function of  $\cos\theta$ ,  $\cos\eta$ , and  $p^*$ :

$$E_\gamma = \frac{m_{\pi^0}^2}{2\Gamma(1-B\cos\theta)(E^* - p^* \cos\eta)}.$$

The detection efficiency  $\epsilon$  is a function of both the photon energy  $E_\gamma$  and the absorber thickness. The latter can be expressed by the angle  $t$  of incidence and the number  $\nu$  of absorber sheets. We used the following empirical formula<sup>6</sup>:

$$\epsilon(E_\gamma, \nu, \cos t) = \left( -0.046 + \frac{0.62}{E_\gamma} \right) \frac{\nu}{|\cos t|} + \left( 0.39 - \frac{4.2}{E_\gamma} \right) \frac{\nu^2}{\cos^2 t},$$

which is valid under the conditions

$$E_\gamma \geq 10 \text{ MeV},$$

$$\frac{\nu}{|\cos t|} \leq 4.5.$$

The angular distribution of the detected photons becomes then

$$\left( \frac{d\sigma_\gamma}{d\Omega} \right)_{\text{det}} = \sum_{l=0}^n a_l P_l(\cos\theta^*) F_l(\cos\theta, \nu, \cos t, p^*),$$

where

$$\begin{aligned} F_l(\cos\theta, \nu, \cos t) &= \int_0^{p_{\text{max}}^*} f(p^*) dp^* \\ &\times \int_{-1}^{+1} \frac{m_{\pi^0}^2 \epsilon(E_\gamma, \nu, \cos t) P_l(\cos\eta) d(\cos\eta)}{2\pi\Gamma^2(1-B\cos\theta)^2 (E^* - p^* \cos\eta)^2}. \end{aligned}$$

We then took into account the solid-angle dependence by integrating  $(d\sigma_\gamma/d\Omega)_{\text{det}}$  over the length of the target and the surface of the converters, and summing over the number of absorbers traversed. The counting rate  $G_\gamma^i$  of the  $i$ th detector is then

$$G_\gamma^i = n_\pi N \sum_\nu \int_{(\Omega_i)} \left( \frac{d\sigma_\gamma}{d\Omega} \right)_{\text{det}} d\Omega,$$

where  $N$  is the number of protons per  $\text{cm}^2$  of the target.  $n_\pi$  is the number of incident pions, and  $(\Omega_i)$  the mean solid angle of the  $i$ th detector. We obtain, finally, five equations of the form

$$G_\gamma^i = \sum_{l=0}^n a_l Q_{il},$$

where

$$Q_{il} = N \sum_\nu \int_{(\Omega_i)} d\Omega_i P_l(\cos\theta_i^*) F_{li}(\cos\theta_i, \nu, \cos t_i).$$

<sup>10</sup> J. P. Merlo, Laboratoire de Physique Corpusculaire à Haute Energie Report No. 62-10, Centre d'Etudes Nucléaires de Saclay (unpublished).

The coefficients  $a_i$  were calculated by the least-squares method. If  $n_0$  is the degree corresponding to the best fit, the total cross section for the photon production (uncorrected) is

$$\sigma_\gamma = 8\pi a_0^{(n_0)}.$$

The calculation was carried out with two different forms for the spectra  $f(p^*)$ ,  $f_1(p^*)$  having a maximum at a high-momentum value similar and close to the phase-space distribution, and  $f_2(p^*)$  with a maximum at a low-momentum value:

$$f_1(p^*) = \frac{12}{p_{\max}^{*4}} p^{*2} (p_{\max}^* - p^*),$$

$$f_2(p^*) = \frac{12}{p_{\max}^{*4}} p^* (p_{\max}^* - p^*)^2.$$

Both cases yield identical results within the statistical error.

### B. Corrections to the Total Cross Section $\sigma_\gamma$

Two effects, double scattering and neutrons, cause an apparent increase in the cross section  $\sigma_\gamma$ . The double scattering occurs if, after the incident  $\pi^+$  meson has interacted in the target, one of the secondary particles interacts in the hydrogen, in the walls of the target, or in the matter of the scintillators, producing photons. This effect was calculated and found to amount to 3% of the total cross section. The effect due to the neutrons was estimated in the following manner: It was assumed that the lead and carbon absorbers have the same efficiency for neutrons, which is correct only for neutrons of 90 MeV.<sup>11</sup> Calculating the average efficiency of lead

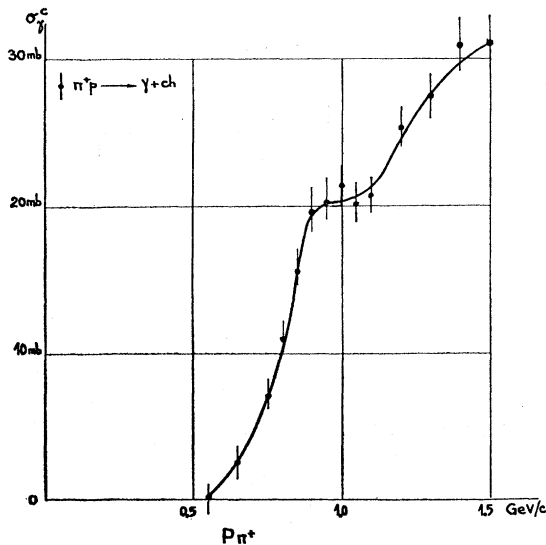


FIG. 7. Weighted cross section of the photon production in  $\pi^+ p$  interactions.

<sup>11</sup> J. Hadley and H. York, Phys. Rev. 80, 345 (1950).

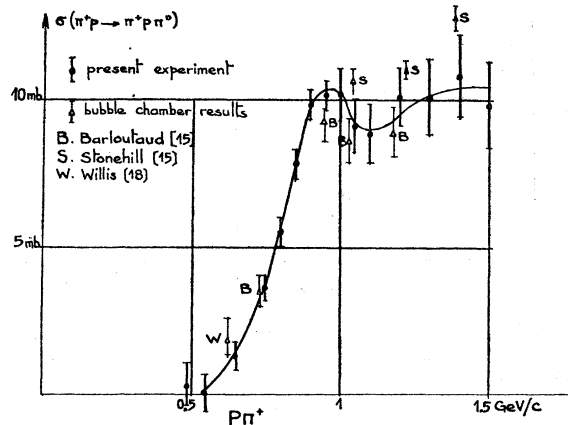


FIG. 8. Cross section for the  $\pi^+ p \rightarrow \pi^+ p \pi^0$  reaction. The solid line is an arbitrary fit to our data.

and carbon using the experimental spectrum of neutrons from the  $\pi^+ p \rightarrow \pi^+ \pi^+ n$  reaction, we found a difference in the efficiency of the two types of absorbers smaller than  $3 \times 10^{-2}$  (see Ref. 6). Since the cross section for the  $\pi^+ \pi^+ n$  is smaller than 5 mb below 1.5 GeV/c, we have a correction which attains, at most 0.15 mb.

A loss in the photon count for the  $\pi^+ n \rightarrow \pi^+ p \pi^0$  reaction may occur for the following reasons:

(a). The terminal counter  $F$  is triggered either by a charged secondary or by a  $\delta$  ray produced by the incident  $\pi$  meson in the target.

(b). The counter  $C$  is not triggered, either because the charged secondaries are emitted through the gaps of the counter  $C$  (necessary for the passage of the beam and the target feed) or because their energy is insufficient to leave the target.

(c). Photons are lost by internal conversion or by conversion in the hydrogen or in the surrounding matter.

(d). A  $\gamma$  telescope does not record the photon because a charged particle passes through its anticoincidence scintillator.

(e). Both photons of the  $\pi^0$  decay are emitted towards the same telescope and therefore cannot produce more than one output pulse.

The corresponding geometric corrections (a), (b), (d), and (e) were calculated using the statistical model.<sup>12</sup> For a few points these corrections were also calculated using the actual angular distributions and spectra of the secondaries as known from bubble chamber measurements.<sup>13</sup> We found that the error of the geometric corrections was negligible as compared with the error  $\Delta\sigma_\gamma$ . Between 0.5 and 1.5 GeV/c the geometric corrections vary from 7 to 8%.

<sup>12</sup> J. P. Merlo, Laboratoire de Physique Corpusculaire à Haute Energie Report No. 62-11, Centre d'Etudes Nucléaires de Saclay, Seine et Oise (unpublished).

<sup>13</sup> R. Barloutaud (private communication).

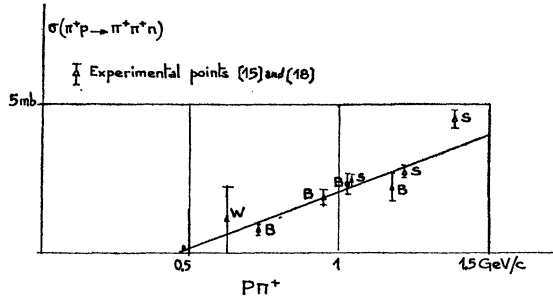


FIG. 9. Cross section for the  $\pi^+p \rightarrow \pi^+\pi^+n$  reaction. The solid line represents this cross section as deduced from our experiment (see IIIC).

Correction (c) was estimated in the following way:

It was assumed that 1.2% of the  $\pi^0$  mesons disintegrate producing a Dalitz pair; we lose thus 0.6% of the photons by internal conversion. For the external conversion, we have calculated the probability that photons of different energy and emitted at different directions, materialize in the target or in the surrounding matter. We found an average probability of  $4 \times 10^{-2}$  that a photon gives an electron pair by external conversion before reaching the photon telescopes.

After applying all the above corrections, we obtain the curve shown in Fig. 7, which gives the energy dependence of the weighted cross section  $\sigma_\gamma^c$  of the photon production in  $\pi^+p$  interactions.

### C. Cross Section for the Reaction $\pi^+p \rightarrow \pi^+p\pi^0$

The different sources of photons in the  $\pi^+p$  reactions below 1.5 GeV/c are as follows:

$$\begin{aligned} \pi^+p &\rightarrow \pi^+p\pi^0 \\ &\pi^+p\eta^0 \\ &K^+\Sigma^+ \\ &\pi^+p\pi^0\pi^0 \\ &\pi^+n\pi^+\pi^0. \end{aligned}$$

$\eta^0$  disintegrates into  $\pi^+\pi^-\pi^0$ ,  $3\pi^0$ , or two photons with a branching ratio 2:3:3,<sup>14</sup> and as a result, produces on the average 3.5 photons. The cross section for the  $\pi^+p\eta^0$  channel is equal to 0.3 mb at<sup>15</sup> 1.27 GeV/c, and we have assumed that it varies linearly beginning from the threshold at 930 MeV/c.

The cross section for the  $\pi^+p \rightarrow K^+\Sigma^+$  reaction is known.<sup>16</sup> In 50% of the cases  $\Sigma^+$  disintegrates into  $p\pi^0$  while the  $K^+$  having a lifetime of  $1.2 \times 10^{-8}$  sec leaves

<sup>14</sup> P. Bastien, J. Berge, O. Dahl, M. Ferro Luzzi, D. Miller, J. Murray, A. H. Rosenfeld, and M. Watson, *Phys. Rev. Letters* **8**, 114 (1962); M. Chretien, F. Bulos, H. R. Crouch, R. E. Lanou, J. T. Massimo *et al.*, *ibid.* **9**, 127 (1962).

<sup>15</sup> G. W. Tautfest, R. B. Willmann, M. C. Foster, M. L. Good, R. P. Matson, and M. W. Peters, *Bull. Am. Phys. Soc.* **7**, 468 (1962).

<sup>16</sup> D. Stonehill, C. Baltay, H. Courant, W. Fickinger, E. C. Fowler *et al.*, *Phys. Rev. Letters* **6**, 624 (1961). R. Barloutaud (private communication).

the target without decay. The reaction  $\pi^+p \rightarrow K^+\Sigma^+$  gives, then, on the average, one photon.

The Yale group<sup>16</sup> has measured the cross section for the double production reactions between 1.04 and 1.39 GeV/c.

We have calculated the correction

$$\left[ \{3.5\sigma(\pi^+p\eta^0) + \sigma(K^+\Sigma^+) + 4\sigma(\pi^+p\pi^0\pi^0) + 2\sigma(\pi^+n\pi^+\pi^0)\} \right]$$

at the point for which we know the corresponding cross sections, and we have extrapolated it to 1.5 GeV/c. The correction was subtracted from  $\sigma_\gamma^c$ . The remainder represents, then, the cross section for the production of photons via the  $\pi^+p\pi^0$  reaction. To obtain the cross section of the latter reaction it is sufficient to divide by two. The result is shown in Fig. 8.

The knowledge of  $\pi^+p\pi^0$  cross section, together with  $\sigma_{\text{tot}}(\pi^+p)$ ,<sup>17</sup> the elastic cross section,<sup>18</sup> and the multiple-production cross section, enables us to calculate the cross section for the  $\pi^+p \rightarrow \pi^+\pi^+n$  reaction

$$\sigma(\pi^+\pi^+n) = \sigma_{\text{tot}} - \sigma_{\text{el}}(\pi^+p) - \sigma(\pi^+p\pi^0) - \sigma_{\text{mult. prod.}}$$

We obtain a result (Fig. 9) which is in a good agreement with the direct measurements known so far.<sup>19</sup>

### IV. CONCLUSION

The cross section for the  $\pi^+p\pi^0$  reaction increases very rapidly, similarly to that for the  $\pi^-p \rightarrow \pi^-\pi^+n$  reaction<sup>2</sup> but with a shift of 200 MeV/c, and attains a maximum of 10 mb at 1 GeV/c. This difference can be explained within the framework of the peripheral model as being due to the absence of the state  $I=0$  in the system ( $\pi^+\pi^0$ ) while it seems that at the threshold of the single  $\pi$  production such a state is predominant.<sup>20</sup> The cross section for the  $\pi^+\pi^+n$  reaction varies linearly

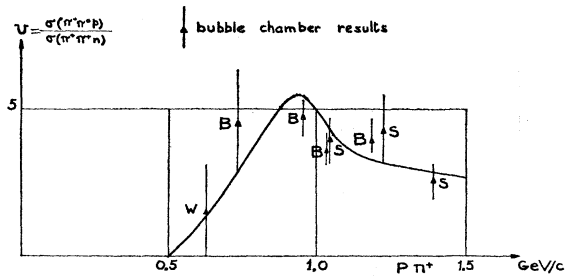


FIG. 10. Variation of the ratio  $\sigma(\pi^+\pi^+p)/\sigma(\pi^+\pi^+n)$  as a function of the incident  $\pi^+$  momentum. The solid line represents this ratio as deduced from our experiment (see IIIC) with an uncertainty of the order of 20%.

<sup>17</sup> J. C. Brisson, J. F. Detoeuf, P. Falk-Vairant, G. Valladas, and L. Van Rossum, *Nuovo Cimento* **19**, 210 (1961).

<sup>18</sup> J. A. Helland, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and C. D. Wood, in *Proceedings of the International Conference on High-Energy Physics, CERN, 1962* (CERN, Geneva, 1962), p. 3.

<sup>19</sup> W. J. Willis, *Phys. Rev.* **116**, 753 (1959). J. Kirz, J. Schwartz, and R. D. Tripp, University of California Radiation Laboratory Report No. UCRL-9941 (unpublished); *Phys. Rev.* **126**, 763 (1962); Ref. 16.

<sup>20</sup> H. J. Schnitzer, *Phys. Rev.* **125**, 1059 (1962).

with the momentum as one would expect to be the case in the absence of the  $\pi N$  ( $\frac{3}{2} \frac{3}{2}$ ) isobar and the state  $I=1$  of the dipion.

We have determined (Fig. 10) the variation of the branching ratio  $U = \sigma(\pi^+ p \pi^0) / \sigma(\pi^+ \pi^+ n)$ . No model predicts the observed behavior. However, the slow decrease above 800 MeV/c agrees with the predictions of Olsson and Yodh.<sup>21</sup>

Other remarkable features of our results are the following:

(1). The peak at 1 GeV/c in the cross section  $\pi^+ p \pi^0$  corresponds to the "shoulder" in the total cross section and is much more marked in this inelastic reaction than in the elastic cross section. The energy is close to the threshold for the reaction  $\pi^+ p \rightarrow \rho^+ p$ .

<sup>21</sup> M. Olsson and G. B. Yodh, Phys. Rev. Letters **10**, 353 (1963).

(2). The reactions  $\pi^+ p \rightarrow \pi^+ p \pi^0$  and  $\pi^- p \rightarrow \pi^- \pi^+ n$  are, among the five reactions of single-pion production, the only ones which contain a pure state with  $I=\frac{3}{2}$  of the  $(\pi N)$  system, and, in both of them, an  $I=1$  state of the di-pion is possible. The behavior of the two channels is rather similar, in spite of the shift of 200 MeV/c. The corresponding cross sections increase rapidly and attain a value of 10 mb around 1 GeV/c.

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## Electromagnetic Violation of Conservation of Vector Current\*

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The electromagnetic violation of vector current conservation is considered and the strong interaction renormalization of the  $\beta$ -decay coupling constant  $G_V$  in the presence of electromagnetism is estimated. It is shown that the existing discrepancy between the  $G_V$  and the  $\mu$ -decay coupling constant  $G_\mu$  may be due to such a renormalization. In this demonstration, use is made of a hypothetical scalar particle with isotopic spin unity. This particle has been used as an auxiliary means and the final results do not depend on the parameters of such a particle. It is shown that this particle has interactions with other particles only if the electromagnetic mass splitting of the isotopic multiplets is not neglected. Other properties of this particle are discussed.

**I**N the local four-Fermion universal theory of weak interactions, the  $\beta$ -decay coupling constant  $G_V$  and that for the  $\mu$  decay  $G_\mu$  should be equal in the absence of radiative effects since, according to the conserved vector current hypothesis, there is no renormalization<sup>1</sup> of  $G_V$  due to strong interactions when electromagnetism is neglected. But it is well known that there is a discrepancy<sup>2</sup> of about 2% between  $G_V$  and  $G_\mu$  after elimination of the radiative effects. In calculating these radiative effects,<sup>2,3</sup> however, only the simple perturbative diagrams were considered. Thus the change in the strong interaction renormalization of  $G_V$  due to radiative effects has not yet been calculated. That is to say, whereas

previously in the absence of electromagnetism the whole set of strong interaction graphs which renormalize  $G_V$  summed up to unity, now, in the presence of electromagnetism, the strong interaction renormalization factor is no longer unity. Rather, we should expect  $G_V = G_\mu + \delta G_V$ , where<sup>4</sup>  $\delta G_V = O(\alpha)$ ,  $\alpha$  being the fine-structure constant. The purpose of this paper is to estimate  $\delta G_V$  due to the strong interaction renormalization of  $G_V$  in the presence of electromagnetism and to see whether such an effect can remove the discrepancy between  $G_V$  and  $G_\mu$ . We shall demonstrate that it is

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<sup>1</sup> R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

<sup>2</sup> L. Durand, III, L. F. Landowitz, and R. B. Marr, Phys. Rev. **130**, 1188 (1963). In this paper, this discrepancy is thoroughly discussed and references to experimental papers and other relevant literature regarding the radiative corrections are given.

<sup>3</sup> R. E. Behrends, R. J. Finkelstein, and A. Sirlin, Phys. Rev. **101**, 866 (1956); S. M. Berman, *ibid.* **112**, 267 (1958); T. Kinoshita and A. Sirlin, *ibid.* **113**, 1652 (1959); S. M. Berman and A. Sirlin, Ann. Phys. (N.Y.) **20**, 20 (1962); N. Chang, Phys. Rev. **131**, 1272 (1963).

<sup>4</sup> R. Behrends and A. Sirlin, Phys. Rev. Letters **4**, 186 (1960) and M. V. Terent'ev, Zh. Eksperim. i Teor. Fiz. **44**, 1320 (1963) [English transl.: Soviet Phys.—JETP **17**, 890 (1963)]. These authors have shown that the effect of the mass differences of isotopic multiplets on the  $G_V$  is at least second order in the magnitude of the relative mass differences. In their work, however, they essentially considered the effect of the insertion of all electromagnetic self-energy parts in the set of strong interaction renormalization graphs. As has been pointed out by Chang in Ref. 3, electromagnetism has more effects than just self-energy change of the internal charge lines and, therefore, the work of Behrends and Sirlin and Terent'ev does not imply that  $G_V - G_\mu \sim G_\mu O(\alpha)$ . This will in fact be born out by our calculations.



